

GROUPS OF ORDER 2048 WITH THREE GENERATORS AND THREE RELATIONS

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ABSTRACT. It is shown that there are exactly seventy-eight 3-generator 2-groups of order 2^{11} with trivial Schur multiplier. We then give 3-generator, 3-relation presentations for forty-eight of them proving that these groups have deficiency zero.

1. Introduction

A finite group is said to have deficiency zero if it has a deficiency zero presentation, namely a presentation with an equal number of generators and relations. A classical fact is that finite groups of deficiency zero have trivial Schur multiplier, for example see [9, p.87]. So the Schur multiplier provides a useful criterion in the search for finite groups of deficiency zero. But the converse is not true since there are many examples of finite groups with trivial Schur multiplier and non-zero deficiency. These

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groups are all non-nilpotent. In fact, it is a long-standing question about finite p -groups with trivial Schur multiplier whether they have deficiency zero, see [14, Question 12]. In [6], the authors prove a number of p -groups have deficiency zero and give explicit presentations for them with an equal number of generators and relations. It is noted in [6] that there are no 3-generator 2-groups of order less than 2^9 having trivial Schur multiplier and there exist exactly two such groups of order 2^9 . Moreover in [4] we see that there are exactly eighteen 3-generator 2-groups of order 2^{10} with trivial Schur multiplier all having deficiency zero. Many finite d -generator, d -relation groups are known for $d = 1, 2, 3$. Trivial examples are the finite cyclic groups with $d = 1$ and the symmetric group of degree 3 with $d = 2$. In fact many examples with $d = 2$ have been given by several authors. Examples of finite groups with $d = 3$ are infrequent, see [8] and the references therein. It might be worth noting that there are no known examples of finite groups with $d = 4$ and finite nilpotent 4-generator groups require at least 5 defining relations by a celebrated theorem of Golod-Shafarevich. Such groups have been constructed in [5, 7], of orders $2^{14}, 2^{16}, 2^{17}, 2^{18}$ and 2^{19} .

In this paper, using computational methods we show that there are exactly seventy-eight 3-generator 2-groups of order 2^{11} with trivial Schur multiplier. We then give 3-generator, 3-relation presentations for forty-eight of them proving that these groups all have deficiency zero.

Our notation is standard. \mathbb{Z}_n is the cyclic group of order n . The direct product of ℓ copies of \mathbb{Z}_n is denoted by \mathbb{Z}_n^ℓ . The Schur multiplier of the group G is denoted by $M(G)$. We write `SmallGroup(n, m)` for the m th group of order n as quoted in the "Small Groups" library in GAP [13].

2. Method

In this section our first step is to determine all 3-generator groups of order 2^{11} with trivial Schur multiplier. Then our second step is to show that some of these groups have deficiency zero.

We describe below a method that enables one to determine 3-generator groups of order 2^{11} having trivial Schur multiplier. We use the computer algebra systems GAP [13] and MAGMA [2] which contain a data library “Small Groups” providing access to the descriptions of the groups of order at most 2000 except 2^{10} , prepared by Besche *et al* [1]. Following [6], our main strategy is to determine some particular extensions, called descendants, of specified 3-generator 2-groups G , $|G| \leq 2^9$, in the hope of finding 2-groups of order 2^{11} with trivial Schur multiplier. To do this we will use the following theorems.

Theorem 2.1. [10, Theorem 3.2.1] *Suppose that N is a normal subgroup of a finite group G . If F is a free group of finite rank, R is a normal subgroup of F for which $G \cong F/R$ and S is a normal subgroup of F for which SR/R corresponds to N , then there is an exact sequence*

$$1 \rightarrow \left(\frac{R \cap [F, S]}{([F, R] \cap [F, S])} \right) \rightarrow M(G) \rightarrow M\left(\frac{G}{N}\right) \rightarrow \frac{(N \cap G')}{[N, G]} \rightarrow 1.$$

Theorem 2.2. [10, Corollary 3.2.2] *Suppose that N is a normal subgroup of a finite group E . If $M(E) = 1$, then $M\left(\frac{E}{N}\right) \cong \frac{(N \cap E')}{[N, E]}$.*

Recall that the lower exponent- p central series of G is a descending series of subgroups defined recursively by $P_0(G) = G$, $P_{i+1}(G) = [P_i(G), G]P_i(G)^p$ for $i \geq 0$. If c is the smallest integer such that $P_c(G) = 1$, then G has exponent- p class c . A group E is said to be a descendant of a finite d -generator p -group G with exponent- p class c if the quotient $E/P_c(E)$ is isomorphic to G . A group is called an immediate descendant of G if it is a descendant of G and has exponent- p class $c+1$. Both

GAP and MAGMA compute the lower exponent- p central series of a finite group using the p -quotient algorithm described in [11] and are able to construct all immediate descendants of a given p -group by the p -group generation algorithm [12].

Now we determine all 3-generator 2-groups E of order 2^{11} with trivial Schur multiplier.

Lemma 2.3. *Let E be a 3-generator 2-group of order 2^{11} with trivial Schur multiplier. Then E is an immediate descendant of a 3-generator group G of order 2^n ($n \leq 10$) which satisfies $M(G) \cong \mathbb{Z}_2^\ell$, where $0 \leq \ell \leq 11 - n$.*

Proof. Suppose that E has exponent- p class $c + 1$. Using Theorem 2.2, with $N = P_c(E)$, we have $M(E/P_c(E)) \cong (P_c(E) \cap E')/[P_c(E), E]$. By our hypothesis on the class of E , we observe that $P_c(E)^2 = 1$, from which we conclude that $P_c(E)$ is an elementary abelian 2-group and that $M(E/P_c(E)) \cong P_c(E) \cap E'$. Now the group $G := E/P_c(E)$ is a 3-generator group with $M(G) \hookrightarrow P_c(E)$ and so $|M(G)| \leq |E|/|G|$. \square

The above lemma reduces the number of groups that need to be considered dramatically. We use MAGMA and GAP to construct all immediate descendants E of such groups G and rule out those having non-trivial Schur multiplier. Since all groups G of order 2^n ($n \leq 9$) are available in GAP, first we determine all immediate descendants of groups G which satisfy $M(G) \cong \mathbb{Z}_2^\ell$, where $0 \leq \ell \leq 11 - n$. In the list below there are forty 3-generator groups of order 2^{11} with trivial Schur multiplier with the above property. We use the notation $[n, m, k]$ for the group E , where E is the k th immediate descendant of the group $G = \text{SmallGroup}(n, m)$.

[512, 6489, 2], [512, 6489, 3], [512, 6490, 2], [512, 6490, 3], [512, 9113, 4],
 [512, 9113, 5], [512, 9114, 4], [512, 9114, 5], [512, 9121, 4], [512, 9121, 5],
 [512, 9122, 4], [512, 9122, 5], [512, 9137, 4], [512, 9137, 5], [512, 9146, 4],

[512, 9146, 5] , [512, 12397, 4], [512, 12397, 5], [512, 12398, 4],
 [512, 12398, 5], [512, 12399, 4], [512, 12399, 5], [512, 12400, 4],
 [512, 12400, 5], [512, 12401, 4], [512, 12401, 5], [512, 12402, 4],
 [512, 12402, 5], [512, 12403, 2], [512, 12403, 3], [512, 12404, 2],
 [512, 12404, 3], [512, 12413, 4], [512, 12413, 5], [512, 12414, 4],
 [512, 12414, 5], [512, 12423, 4], [512, 12423, 5], [512, 12424, 4],
 [512, 12424, 5].

Now by Lemma 2.3, we have to consider 3-generator groups G of order 2^{10} with $M(G) \cong \mathbb{Z}_2^\ell$, where $0 \leq \ell \leq 1$. All 3-generator groups of order 2^{10} with trivial Schur multiplier are classified in [4]. By using GAP we see that there is no immediate descendant of order 2^{11} of these eighteen groups of order 2^{10} . Since groups of order 2^{10} are not available in GAP, we state the following theorem to construct groups of order 2^{10} with Schur multiplier of order 2.

Theorem 2.4. *Let E be a 3-generator 2-group of order 2^{10} with $M(E) \cong \mathbb{Z}_2$. Then E is an immediate descendant of a 3-generator group G of order 2^n with $n \leq 9$ which satisfies either $M(G) \cong \mathbb{Z}_2^\ell$, $0 \leq \ell \leq 11 - n$ or $M(G) \cong \mathbb{Z}_4 \times \mathbb{Z}_2^\ell$, $0 \leq \ell \leq 9 - n$.*

Proof. Suppose that E has exponent- p class $c + 1$. By Theorem 2.1, we have the following exact sequence: $1 \rightarrow (R \cap [F, S])/([F, R] \cap [F, S]) \rightarrow M(E) \xrightarrow{\alpha} M(E/P_c(E)) \xrightarrow{\beta} (P_c(E) \cap E')/[P_c(E), E] \rightarrow 1$, where F is a free group of finite rank, R is a normal subgroup of F for which $E \cong F/R$ and S is a normal subgroup of F for which SR/R corresponds to $P_c(E)$. On setting $G = E/P_c(E)$ we see that $M(G)/\text{Ker}\beta \cong P_c(E) \cap E'$ and $M(E)/\text{Ker}\alpha \cong \text{Ker}\beta$. Therefore $\text{Ker}\beta = 1$ or $\text{Ker}\beta \cong \mathbb{Z}_2$ since $M(E) \cong \mathbb{Z}_2$. Now since $M(G)/\text{Ker}\beta \hookrightarrow P_c(E)$ and $P_c(E)$ is elementary abelian, we deduce that $|M(G)| \leq 2|P_c(E)|$ and $M(G)$ is either elementary abelian or $M(G) \cong \mathbb{Z}_4 \times \mathbb{Z}_2^\ell$. \square

Now it only remains to determine all immediate descendants of groups of order 2^{10} with Schur multiplier of order 2. In the list below there are thirty-eight groups of order 2^{11} with trivial Schur multiplier with the above property. We use the notation $[n, m, k, t]$ for the group E , where E is the t th immediate descendant of the group L such that L is the k th immediate descendant of $G = \text{SmallGroup}(n, m)$, in fact L is a 3-generator group of order 2^{10} with Schur multiplier of order 2.

[512, 53479, 3, 1], [512, 53479, 3, 2], [512, 53480, 1, 1], [512, 53480, 1, 2],
 [512, 53480, 2, 1], [512, 53480, 2, 2], [256, 2525, 8, 1], [256, 2525, 8, 2],
 [256, 2525, 9, 1], [256, 2525, 9, 2], [256, 2528, 5, 1], [256, 2528, 5, 2],
 [256, 2528, 6, 1], [256, 2528, 6, 2], [256, 3638, 8, 1], [256, 3638, 8, 2],
 [256, 3639, 8, 1], [256, 3639, 8, 2], [256, 3640, 5, 1], [256, 3640, 5, 2],
 [256, 3640, 6, 1], [256, 3640, 6, 2], [256, 3641, 8, 1], [256, 3641, 8, 2],
 [256, 3641, 9, 1], [256, 3641, 9, 2], [256, 3641, 10, 1], [256, 3641, 10, 2],
 [256, 3643, 8, 1], [256, 3643, 8, 2], [256, 3643, 9, 1], [256, 3643, 9, 2],
 [256, 3643, 10, 1], [256, 3643, 10, 2], [256, 2522, 6, 1], [256, 2522, 6, 2],
 [256, 2523, 6, 1], [256, 2523, 6, 2].

The second step is to give 3-generator, 3-relation presentations for the groups obtained in the first step. We used mainly the method described in [7] to find such a presentation for each group G under consideration. Our first attempt towards obtaining such presentations for G was to find several triples of generators for each group. On each generating triple, we computed a presentation $\langle X | R \rangle$ using the relation finding algorithm of Cannon [3] which is available in GAP and MAGMA. Then an attempt was made to find a subset S of R having three elements such that $\langle X | S \rangle$ defines G . In searching for generating triples for each group a small set of group elements was chosen by a knowledge of conjugacy classes and checked for generating triples. This technique was also used in [4] to determine deficiency zero presentations for all 3-generator, 2-groups of order 2^{10} with trivial Schur multiplier. The authors obtained seventeen

deficiency zero presentations from eighteen groups in [4] by this method. It seems that this method is useful to find deficiency zero presentations. Moreover in this paper to find the order of the groups defined by the presentations $\langle X|S \rangle$ as above, we use Knuth-Bendix algorithm in KBMAG package, which was written by Derek Holt [13]. By the above observation we show that forty-eight groups from seventy-eight 3-generator groups of order 2^{11} with trivial Schur multiplier, have deficiency zero.

3. Results

In three tables below we list all 3-generator 2-groups of order 2^{11} with trivial Schur multiplier. In tables 1 and 2 we list forty-eight groups with deficiency zero. Also table 3 give a presentation for the remaining thirty groups with more than three relations in which we show that the above method failed to find a balanced presentation for these groups. Entries of the form $[n, m, k]$ and $[n, m, k, t]$ were described in the previous section. An attempt was made to choose a presentation for each group with a reasonably small length.

Table 1

Group No.	Relators	$[n, m, k]$
#1	$b^{-1}acabc^{-1}, c^2ab^2a, ab^{-1}cacb^{-1}$	[512, 6489, 2]
#2	$bca^{-1}c^{-1}ba, bac^{-1}b^{-3}ca, cbca^{-1}c^2a^{-1}b$	[512, 6489, 3]
#3	$ba^{-1}c^{-2}ba, a^2cbcb^{-1}, bcbaca^{-1}$	[512, 6490, 2]
#4	$bac^{-1}b^{-1}ca, b^2cac^{-1}a, b^{-1}cbc^3a^{-2}$	[512, 6490, 3]
#5	$b^2ca^{-1}ca, c^{-1}a^2b^{-1}cb, ba^{-1}b^{-1}c^4a$	[512, 9113, 4]
#6	$ba^{-1}b^3a, b^{-1}c^3bc^{-1}, a^3b^{-1}cba^{-1}c^{-1}$	[512, 9113, 5]
#7	$bca^{-1}b^{-1}ca, b^3cbc, a^3b^{-1}a^{-1}cbc^{-1}$	[512, 9114, 4]
#8	$cbcb^{-1}, ba^{-1}b^3a, a^3c^3a^{-1}c^{-1}$	[512, 9114, 5]
#9	$ba^{-1}c^{-1}bc^{-1}a, ab^{-2}cac^{-1}, a^2c^3bc^{-1}b^{-1}$	[512, 9122, 4]
#10	$ba^{-1}c^{-1}bc^{-1}a, ca^{-1}c^{-1}b^2a^{-1}, a^2c^{-1}bc^3b^{-1}$	[512, 9122, 5]
#11	$a^{-1}cbcab^{-1}, b^3a^{-1}ba, cac^3a^{-3}$	[512, 9137, 4]

Table 1

Group No.	Relators	$[n, m, k]$
#12	$a^{-1}cbcab^{-1}, b^3a^{-1}ba, a^3ca^{-1}c^3$	[512, 9137, 5]
#13	$cbcb^{-1}, a^{-1}ba^{-2}b^2a^{-1}b, c^3aca^{-3}$	[512, 9146, 4]
#14	$cbcb^{-1}, a^{-1}ba^{-2}b^2a^{-1}b, a^3c^3a^{-1}c$	[512, 9146, 5]
#15	$a^3bab, acab^{-1}c^{-1}b, b^2cac^{-3}a$	[512, 12397, 4]
#16	$a^3bab, acab^{-1}c^{-1}b, b^2c^{-1}ac^3a$	[512, 12397, 5]
#17	$baba^{-1}, a^2ca^{-1}cbab^{-1}, b^3c^3b^{-1}c^{-1}$	[512, 12398, 4]
#18	$cac^{-1}a, b^{-1}a^{-1}ba^2b^2a, bac^{-3}bca$	[512, 12398, 5]
#19	$bab^{-1}a, a^{-1}c^3ac^{-1}, a^3c^{-1}abc^{-1}b^{-3}$	[512, 12400, 4]
#20	$bab^{-1}a, a^{-1}c^3ac^{-1}, a^2b^{-1}a^{-1}ca^{-1}bcb^{-2}$	[512, 12400, 5]
#21	$cbc^{-1}b, ac^3a^{-1}c^{-1}, ba^{-1}ba^2b^2a$	[512, 12401, 4]
#22	$cbc^{-1}b, a^{-1}c^3ac^{-1}, ba^{-1}ba^2b^2a$	[512, 12401, 5]
#23	$cbc^{-1}b, ba^{-1}b^3a^{-1}, a^3cac^{-3}$	[512, 12402, 4]
#24	$ab^{-3}ab^{-1}, bc^{-3}bc^{-1}, a^2bca^{-1}ca^{-1}b^{-1}$	[512, 12404, 2]
#25	$bab^3a, cbc^3b, bacacb^{-1}a^{-2}$	[512, 12404, 3]
#26	$ba^{-1}bc^2a, ac^2bab^{-1}, (ca)^2cbcb^{-1}$	[512, 12413, 5]
#27	$b^{-1}a^{-1}b^3a, ca^{-1}cbab^{-1}, a^2c^{-2}(bc^{-1})^2$	[512, 12414, 4]
#28	$cbac^{-1}ab^{-1}, acbca^{-1}b^{-1}, a^2c^{-1}a^{-1}bc^{-1}ab$	[512, 12424, 4]

Table 2

Group No.	Relators	$[n, m, k, t]$
#29	$cbc^{-1}b, a^3b^{-1}c^{-1}ba^{-1}c, a^3bc^{-1}a^{-1}c^{-1}b$	[512, 53480, 2, 1]
#30	$cbc^{-1}b, a^{-3}bcacb, a^3c^{-1}a^{-1}cb^2$	[512, 53480, 2, 2]
#31	$cbcb^{-1}, ba^{-1}c^{-1}bca, a^2b^{-1}acbac$	[256, 2528, 5, 1]
#32	$a^2c^2, bcb^{-1}ac^{-1}a, bacb^3ca^{-1}$	[256, 2528, 5, 2]
#33	$a^2(bc)^2, bcb^{-1}aca^{-1}, a^3c^{-2}bab^{-1}$	[256, 2528, 6, 1]
#34	$ba^{-1}cb^{-1}ca, cb^2ca^{-2}, a^3bcba^{-1}c^{-1}$	[256, 2528, 6, 2]
#35	$c^{-1}bcb, ac^{-1}ac^{-1}b^2, a^2ca^{-1}bc^{-1}ab$	[256, 3640, 5, 1]
#36	$ba^{-1}cbc^{-1}a, ac^3a^{-1}c^{-1}, b^{-3}c^{-1}bca^2$	[256, 3640, 5, 2]
#37	$a^{-1}b^3ab^{-1}, b^{-1}a^{-1}cab, c^{-1}ac^{-3}bab$	[256, 3640, 6, 1]

Table 2

Group No.	Relators	$[n, m, k, t]$
#38	$a^{-1}b^3ab^{-1}, b^{-1}a^{-1}cab, c^{-2}ac^{-1}bcab$	[256, 3640, 6, 2]
#39	$a^{-1}cba^{-1}c^{-1}b, b^3cb^{-1}c^{-1}, a^3b^{-1}c^2ab^{-1}$	[256, 3641, 10, 1]
#40	$caca^{-1}, a^{-1}b^3ab^{-1}, acabc^{-3}b$	[256, 3641, 10, 2]
#41	$cac^3a, bcba^{-1}b^{-2}ca, babca^{-1}c^{-1}a^{-2}$	[256, 3643, 8, 1]
#42	$ac^{-3}ac^{-1}, bcba^{-1}b^{-2}ca, babca^{-1}c^{-1}a^{-2}$	[256, 3643, 8, 2]
#43	$ac^{-3}ac^{-1}, a^{-1}cbcb^2a^{-1}b^{-1}, babca^{-1}c^{-1}a^{-2}$	[256, 3643, 10, 1]
#44	$cac^3a, b^3cb^{-1}a^{-1}ca, babca^{-1}c^{-1}a^{-2}$	[256, 3643, 10, 2]
#45	$a^2(ab^{-1})^2, bc^3b^{-1}c^{-1}, a^3b^2cac^{-1}$	[256, 2522, 6, 1]
#46	$a^2(ab^{-1})^2, bc^3b^{-1}c^{-1}, a^3c^{-1}ab^2c$	[256, 2522, 6, 2]
#47	$ca^{-1}bca^{-1}b^{-1}, cab^{-2}ca^{-1}, a^2c^2(ba)^2$	[256, 2523, 6, 1]
#48	$a^2(ab^{-1})^2, b^{-1}c^3bc^{-1}, a^3cabcb$	[256, 2523, 6, 2]

Table 3

Group No.	Relators	$[n, m, k, t]$
#49	$bac^{-1}b^{-1}ca, ba^{-1}b^3a, ab^5a^{-1}b^{-1}, a^2bcb^3c^{-1},$ $cabcb^{-3}ba$	[512, 9121, 4]
#50	$bac^{-1}b^{-1}ca, ba^{-1}b^3a, ab^5a^{-1}b^{-1}, a^2bcb^3c^{-1},$ $c^{-1}bc^{-2}acab$	[512, 9121, 5]
#51	$a^3b^{-1}a^{-1}b, ac^3a^{-1}c^{-1}, a^5bab^{-1}, a^2c^2a^{-2}c^{-2},$ $aca^{-1}bcb^{-3}$	[512, 12399, 4]
#52	$bc^3b^{-1}c^{-1}, b^3a^3b^{-1}a^{-1}, aca^{-1}c^{-1}a^{-1}cac^{-1},$ $ab^{-1}c^{-1}a^{-1}cb^3, a^2cac^{-2}ac$	[512, 12399, 5]
#53	$bab^3a, b^2cb^2c^{-1}, a^{-1}c^3ac^{-1}, bc^{-1}bca^{-4},$ $(bc)^2(bc^{-1})^2$	[512, 12402, 5]
#54	$a^3b^{-1}a^{-1}b, a^5bab^{-1}, a^3c^{-1}ac^3,$ $a^2c^2a^{-2}c^{-2}, babcb^{-1}b^{-1}cba$	[512, 12403, 2]
#55	$a^3b^{-1}a^{-1}b, a^5bab^{-1}, a^3c^{-3}ac,$ $a^2c^2a^{-2}c^{-2}, bab^2cb^{-1}c^{-1}a$	[512, 12403, 3]

Table 3

Group No.	Relators	$[n, m, k, t]$
#56	$ba^{-1}bc^2a, ac^2bab^{-1}, b^2cb^2c^{-1},$ $cacb^{-1}c^{-1}bc^{-1}a$	[512, 12413, 4]
#57	$bcb^{-1}ca^{-2}, a^2b^{-1}cbc, a^{-1}c^3ac^{-1},$ $a^3b^{-3}a^{-1}b$	[512, 12414, 5]
#58	$ac^2ab^2, c^2a^2b^{-2}, a^{-1}cbcab,$ $c^{-2}ac^{-1}bca^{-1}b, cbaca^{-1}bc^{-1}a^{-1}c^{-1}ba^{-1}b^{-1}$	[512, 12423, 4]
#59	$ac^{-1}abc^{-1}b^{-1}, b^{-1}cbaca, cbc^{-3}b,$ $a^3b^{-1}cac^{-1}b^{-1}$	[512, 12423, 5]
#60	$bcbc^{-1}a^{-2}, b^2c^{-1}aca^{-1}, b^{-1}ca^{-1}cba,$ a^2c^3bcb	[512, 12424, 5]
#61	$b^2ab^2a^{-1}, c^2bc^2b^{-1}, (bc)^2ac^{-1}a^{-1}c,$ $cbcab a^{-3}, a^3cb^{-1}a^{-1}bc^{-1}, a^2c^7b^{-1}c^{-1}b$	[512, 53479, 3, 1]
#62	$a^2ca^{-2}c^{-1}, bacbc^{-1}a, bca^{-1}c^{-1}ba,$ $acabc^{-1}b^{-1}, c^2bc^2b^{-1}, b^{16}ca^{-2}c$	[512, 53479, 3, 2]
#63	$cab^2c^{-1}a^{-3}, cba^{-1}cb^{-1}a^3, a^2c^{-1}b^{-1}a^{-1}cab,$ $b^{-2}cbaba^{-1}c, a^{-1}(bc)^2ac^2$	[512, 53480, 1, 1]
#64	$c^{-1}a^{-1}c^{-1}bab, b^{-1}c^2aba^{-1}, a^2ba^2b^{-1},$ $a^2ca^2c^{-1}, bc^3bca^2c^2$	[512, 53480, 1, 2]
#65	$c^4, a^2ba^2b^{-1}, b^{-1}cacba, acab^{-1}c^{-1}b,$ $b^5c^{-1}a^{-1}cb^{-1}a$	[256, 2525, 8, 1]
#66	$c^4, a^2ba^2b^{-1}, b^{-1}cacba, acab^{-1}c^{-1}b,$ $b^5cb^{-1}aca^{-1}$	[256, 2525, 8, 2]
#67	$bcac^{-1}ba, b^2cb^{-2}c^{-1}, a^3bc^{-1}bac^{-1},$ $a^2c^{-1}bc^3b^{-1}, a^3cab^{-1}cb^{-1}$	[256, 2525, 9, 1]
#68	$c^{-1}b^{-2}cb^2, bc^{-1}acba, b^{-1}acbca^{-3},$ $c^2b^{-1}a^{-1}bc^2a, a^3b^{-1}cb^{-1}ac$	[256, 2525, 9, 2]
#69	$ac^{-2}a^{-1}c^2, ac^{-1}b^{-1}abc, acbac^{-1}b^{-3},$ $c^{-1}a^2cba^2b^{-1}, a^3ca^{-1}b^{-2}c$	[256, 3638, 8, 1]

Table 3

Group No.	Relators	$[n, m, k, t]$
#70	$b^{-1}c^{-1}acba, bc^2b^{-1}c^{-2}, a^4c^2b^{-2},$ $a^3cb^{-1}cab$	$[256, 3638, 8, 2]$
#71	$a^2ca^2c^{-1}, ba^{-1}cb^{-1}ca, b^2cb^{-2}c^{-1},$ $bc^2b^{-1}c^{-2}, a^3b^{-1}ca^{-1}cb^{-1}, b^6c^2$	$[256, 3639, 8, 1]$
#72	$a^2ca^2c^{-1}, bacb^{-1}ca^{-1}, b^2cb^{-2}c^{-1},$ $bc^2b^{-1}c^{-2}, a^{-1}bc^{-1}ba^3c, b^2c^6$	$[256, 3639, 8, 2]$
#73	$a^2ca^2c^{-1}, b^{-1}cbaca^{-3}, cab^2ca^{-3},$ $a^3b^{-1}c^2ab^{-1}$	$[256, 3641, 8, 1]$
#74	$a^2ca^2c^{-1}, b^{-1}cbaca^{-3}, cab^2ca^{-3},$ $ba^{-1}c^2ba^{-3}$	$[256, 3641, 8, 2]$
#75	$a^2b^{-2}, a^2cb^2c^{-1}, a^{-1}c^3ac^{-1},$ $(bc)^2(b^{-1}c^{-1})^2, acbc^{-1}a(b^{-1}a^{-1})^2b^{-1}$	$[256, 3641, 9, 1]$
#76	$a^2b^2, a^2cb^{-2}c^{-1}, a^{-1}c^3ac^{-1},$ $(bc)^2(b^{-1}c^{-1})^2, babc^{-1}aba^{-1}cb^{-1}a^{-1}$	$[256, 3641, 9, 2]$
#77	$a^2ca^2c^{-1}, ab^2a^{-1}b^{-2}, b^3cb^{-1}ca^{-2},$ $bcacb^{-1}a^{-3}, bacbc^{-3}a^{-1}$	$[256, 3643, 9, 1]$
#78	$a^2ca^2c^{-1}, ab^2a^{-1}b^{-2}, a^2cb^{-1}cb^3,$ $bcacb^{-1}a^{-3}, bac^{-1}bc^3a^{-1}$	$[256, 3643, 9, 2]$

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